# Assignment 4

CS 311, Spring 2016
Due: June 2nd, 2016
Courtesy of Dan LeBlanc

**Problem 1**  Use a reduction to show that the language $ALL_{TM}$ is undecidable

$$ALL_{TM} = \{\langle M \rangle \mid \text{where } M \text{ is a TM and } L(M) = \Sigma* \}$$

[10 points]

**Problem 2**  A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing Machine has any useless states. Formulate this problem as a language and show that it is undecidable. [10 points]

**Problem 3**  If $A \leq_m B$ and $B$ is a regular language, does this imply that $A$ is a regular language? Why or why not? [10 points]

**Problem 4**  Prove that the language

$$LOOP_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ loops on all inputs}\}$$

is not recognizable. [10 points]

**Problem 5**  Prove that the 3-SAT problem discussed in class is an element of $NP$ by giving a verifier and a NTM decider that run in poly-time. (Only one of these is required for a proof, but I'd like both for this question.) [10 points]