## Assignment 3

CS 311, Spring 2016 Due May 19, 2016

Courtesy of Dan LeBlanc

**Problem 1** A *Turing machine with left reset* is similar to an ordinary Turing machine, but the transition function has the form

$$\delta: Q \times \Gamma \to Q \times \Gamma \times \{R, RESET\}$$

If  $\delta(q, a) = (r, b, RESET)$ , when the machine is in state q reading an a, the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages. [10 points]

(Hint: Much like previous problems in this class, you'll need to describe some general construction that takes a Turing Machine-with-reset to an ordinary Turing Machine and visa-versa)

**Problem 2** Give the informal descriptions for Turing machines that decide the following languages

- a)  $\{w \mid w \text{ contains twice as many 0s as 1s} \}$  [5 points]
- b)  $\{a + b = c \mid a, b, c \in \{0, 1\} * \text{ and the binary numbers represented by } a \text{ and } b \text{ sum to } c\}$  [5 points]

Problem 3 Show that the Turing-decidable languages are closed under

- a) union [5 points]
- b) intersection [5 points]
- c) complement [5 points]
- d) set difference [5 points]

**Problem 4** Show that the Turing-recognizable languages are closed under concatenation

This problem requires providing constructions that take individual Turing machines and combines them into a new machine that *recognizes* the new language. Remember, this is about Turing-recognizable languages not just decideable so that there's a possibility of non-termination. [10 points]

**Problem 5** For each of the following Turing machine variants determine if the machine is more powerful, equivalent, or less powerful than a single-tape Turing machine. If less powerful describe the class of languages recognized by the machine. Explain your answers.

- a) A Turing Machine that can only make moves to the right and never left. [5 points]
- b) A Turing Machine that can move right one space or move left two spaces. [5 points]
- c) A Turing Machine that never writes to a space on the tape that already contains a symbol. [5 points]