Assignment 2: Context-Free Languages

Computer Science 311
Spring 2016

Due April 28th, 2016

Problem 1 Let A be the complement of the language $\{a^n b^n \mid n \geq 0\}$.

a) Show that A is not regular.

b) Create a context-free grammar that describes A.

Problem 2 Let $G = (V, \Sigma, R, \langle STMT \rangle)$ be the following grammar:

$$\langle STMT \rangle \rightarrow \langle ASSIGN \} \mid \langle IF - THEN \rangle \mid \langle IF - THEN - ELSE \rangle$$
$$\langle IF - THEN \rangle \rightarrow \texttt{if condition then } \langle STMT \rangle$$
$$\langle IF - THEN - ELSE \rangle \rightarrow \texttt{if condition then } \langle STMT \rangle \texttt{ else } \langle STMT \rangle$$
$$\langle ASSIGN \rangle \rightarrow \texttt{a := 1}$$

$$\Sigma = \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a := 1}\}$$
$$V = \{\langle STMT \rangle, \langle IF - THEN \rangle, \langle IF - THEN - ELSE \rangle, \langle ASSIGN \rangle\}$$

Show that G is ambiguous.

Problem 3 The languages $\{0^s 1^s 2^t \mid s, t \geq 0\}$ and $\{0^s 1^t 2^t \mid s, t \geq 0\}$ are context-free. Use those languages to answer the following:

a) Write down CFGs for each of the languages. [5 Points]

b) Use the fact that both languages are context-free to show that the class of context-free languages is *not* closed under intersection. (Use the context-free pumping lemma to show that the resulting language is not context-free.) [5 Points]

c) Use the result from part b and DeMorgan's Laws to show that the class of context-free languages are *not* closed under complement. [5 Points]

Problem 4 Show that if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.